Session 6 : solutions

Exercise 1

1)

$$\frac{\partial f}{\partial m} = 0$$ to find m of equilibrium

$$\frac{\partial f}{\partial m} = a t m + \frac{1}{5!} b m^5$$ (homogeneous setting $\nabla \vec{m} = 0$)

$\Rightarrow$ $m = 0 \leftarrow$ unstable

$$m = \left(5! \frac{a}{b}\right)^{1/4} t^{1/4}$$

2) The Ginzburg criterion for $\langle \delta M^2 \rangle$ remains the same : $\xi^2 \cdot \text{const}$ with $\xi^2 \propto |t|^{-1}$ (the gaussian approx does not change)

$\langle M^2 \rangle$ instead is $\xi^d |t|^{2 \cdot \frac{1}{4}} =$

$= |t|^{-d/2 + \frac{1}{2}}$

Mean-field is correct if

$$\underbrace{|t|^{-d/2 + \frac{1}{2}}}_{\langle M^2 \rangle} \gg \underbrace{|t|^{-1}}_{\langle \delta M^2 \rangle}$$

$\Rightarrow \quad -\frac{d}{2} + \frac{1}{2} < -1$ because $|t| < 1$

$\Rightarrow \quad d > 3$

2) The solution of the diffusion equation can be found by Fourier transforming the equation, and it is

$$P(\vec{x}, t) = \frac{1}{(4\pi D t)^{d/2}} e^{-\frac{\vec{x}^2}{4Dt}}$$

Rescaling the length means

$$\vec{x} \longrightarrow \vec{x}' = \vec{x}/b$$

accordingly, rescaling time means

$$t \longrightarrow t' = t/b$$

Then we have

$$P(\vec{x},t) = \frac{1}{\left(4\pi D \frac{t}{\tau}\tau\right)^{d/2}} e^{-\frac{\vec{x}^2 b^2}{b^2 4D\frac{t}{\tau}\tau}} =$$

$$= \frac{1}{\tau^{d/2}} \frac{1}{(4\pi D t')^{d/2}} e^{-\frac{\vec{x}'^2}{4Dt'}\frac{b^2}{\tau}}$$

If we choose $\tau = b^2$ then

$$P(\vec{x},t) = \frac{1}{b^d} P(\vec{x}',t')$$

just remember that this is a probability density. The probability is

$P(\vec{x},t)\,d\vec{x}$ and $d\vec{x} = b^d \frac{d\vec{x}}{b^d} =$

$$= b^d\, d\vec{x}'$$

and then

$$P(\vec{x},t)\,d\vec{x} = P(\vec{x}',t')\,d\vec{x}'$$

This result tells us that there is a special relation between time and space.

space rescales as the square root of time!